Math 227A: Suggested Exercises for Week 9

The following are suggested exercises for Week 9:

1. Milnor and Stasheff 13A, 13E, 13F, 14A, 14B, 14C, 14D.

2. *The Splitting Principle* Let $p\colon V \to B$ be a complex n-plane vector bundle, and $P(V)$ its projectivization, with $q\colon P(V) \to B$ the fibre bundle map. Show the vector bundle q^*V over $P(V)$ always splits into the direct sum of a line bundle and an $(n-1)$-plane bundle. Conclude that in proving natural relations for the Chern classes, it suffices to compute on a direct sum of line bundles.

3. Let $V \to B$ be an n-plane complex bundle, and for $m < n$ let $\Lambda^m V$ be the vector bundle whose fibre is $\Lambda^m F$ for every fibre F of B. Compute the total Chern class of $\Lambda^m(V)$ in terms of the Chern classes of V.

4. Show that for M a smooth closed oriented manifold, the Euler class $e(M)$ evaluated on the fundamental homology class of M is equal to the algebraic self-intersection number $[TM]^2$ of the zero-section of TM with itself.(Hint: think about Euler characteristic and vector fields.) Use this to show the adjunction formula: If S is a complex surface (so, a real 4-diml manifold) and C is an embedded complex curve of genus $g(C)$, then $2g(C) - 2 = [C]^2 - c_1(S)[C]$, where $[C]^2$ is the algebraic self-intersection number of C inside S and $c_(S)$ is the Chern class of S evaluated on the homology class represented by C.